QUASI-MAJORITY NEIGHBOR SUM DISTINGUISHING EDGE-COLORINGS

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An edge-coloring c of a graph G defines a vertex-coloring $\sigma_c : V(G) \to \mathbb{N}$ by $\sigma_c(v) = \sum_{u \in N_G(v)} c(vu)$ for each $v \in V(G)$. The edge-coloring c is called neighbor sum distinguishing if $\sigma_c(u) \neq \sigma_c(v)$ for every $uv \in E(G)$. A graph is nice if it has no component isomorphic to K_2.

The smallest k for which a neighbor sum distinguishing k-edge-coloring of G exists is denoted by $\chi^e_{\sum}(G)$ and is related to the 1-2-3 Conjecture, proved by Keusch [2]. If the edge-coloring is proper, the smallest k for which such a coloring exists is denoted by $\chi'_{\sum}(G)$ and is connected to a conjecture of Flandrin et al. [1], stating that $\chi'_{\sum}(G) \leq \Delta(G) + 2$ for any nice G distinct from C_5. This conjecture remains open.

We study an edge-coloring that is stronger than that considered in the 1-2-3 Conjecture, and weaker than the edge-coloring proposed by Flandrin et al. A k-edge-coloring of G is called quasi-majority if for every $v \in V(G)$ and every $\alpha \in [k]$, at most $\left\lceil \frac{d(v)}{2} \right\rceil$ edges incident to v are colored with α.

A k-edge-coloring of G is called quasi-majority neighbor sum distinguishing if it is quasi-majority and neighbor sum distinguishing. The smallest k for which G admits such a coloring is denoted by $\chi^{QM}_{\sum}(G)$. We prove that for every nice G we have $\chi^{QM}_{\sum}(G) \leq 12$. This bound improves to 6 for nice bipartite graphs and to 7 for nice graphs with maximum degree at most 4. Moreover, we determine the exact value of $\chi^{QM}_{\sum}(G)$ for complete graphs, complete bipartite graphs, and trees.

We also consider majority neighbor sum distinguishing edge-colorings, where each vertex is incident to at most half of its edges with the same color.

References

[1] E. Flandrin, A. Marczyk, J. Przybyło, J-F. Sacle, M. Woźniak, Neighbour sum distinguishing index. Graphs Combin. 29(5) (2013), 1329–1336.

[2] R. Keusch, A Solution to the 1-2-3 Conjecture. J. Combin. Theory Ser. B 166 (2024) 182–202.