COLOURING-BASED CONNECTIVITY PARAMETERS IN DIGRAPHS

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In this talk, we explore various versions of connectivity in digraphs based on vertex-colourings, arc-colourings, and total-colourings. We focus on different types of rainbow connectivity, where paths are required to avoid repeated colours on arcs, vertices, or both, depending on the colouring model. In particular, we discuss the concepts of rainbow connection, rainbow vertex-connection, and total rainbow connection, along with their associated parameters: the rainbow connection number $\overrightarrow{rc}(D)$, the rainbow vertex-connection number $\overrightarrow{rvc}(D)$, and the total rainbow connection number $\overrightarrow{rtc}(D)$ of a strong digraph D.

We also consider a weaker variant of rainbow colouring known as proper connection, where the requirement is relaxed to demand only that adjacent arcs (or internal vertices, or both arcs and vertices) receive distinct colours. The corresponding parameters in this setting are the proper connection number $\overrightarrow{pc}(D)$, the proper vertex-connection number $\overrightarrow{pvc}(D)$, and the total proper connection number $\overrightarrow{ptc}(D)$. Furthermore, we explore stronger versions of these concepts, such as strong rainbow connection and strong proper connection, where the colouring conditions must be satisfied along shortest paths (geodesics). Another related notion is proper-walk connection, a more flexible model in which properly coloured walks - not necessarily paths - connect every ordered pair of vertices.

The presentation will survey fundamental results concerning these parameters, illustrate the relationships among them, and outline recent advances in the area. Additionally, we will propose new open problems and directions for further research.