## ISOLATION OF CYCLES IN GRAPHS

PETER BORG, DAYLE SCICLUNA

*University of Malta*

e-mail: peter.borg@um.edu.mt, dayle.scicluna.09@um.edu.mt

Since the seminal work of Caro and Hansberg [3] in 2017 introducing graph isolation, a substantial body of research has emerged in this area. Given a graph $G$ and a set $\mathcal{F}$ of graphs, the $\mathcal{F}$-isolation number is the minimum size of a vertex subset $D$ such that $G - N[D]$ contains no subgraph isomorphic to a member of $\mathcal{F}$. Borg [2] proved that the cycle isolation number is at most $\lfloor n/4 \rfloor$ for connected graphs not equal to $C_3$, and this bound is sharp.

Let $\mathcal{C}$ denote the set of all cycles. Zhang and Wu [6] proved that if $G$ is a connected triangle-free $n$-vertex graph that is not a 4-cycle, then the $\mathcal{C}$-isolation number is at most $\lfloor n/5 \rfloor$, and characterized the extremal graphs.

For cycles of fixed length, determining tight bounds remains challenging. For $C_4$, Bartolo, Borg and Scicluna [1] showed that the $C_4$-isolation number is at most $\lfloor n/5 \rfloor$, apart from nine exceptional graphs. Edge-based bounds were provided by Zhang and Wu [5], who proved that triangle-free graphs with $m$ edges satisfy $\iota(G, \mathcal{C}) \leq (m+1)/6$. Wei, Zhang, and Zhao [4] further showed that for $G \not\cong C_4$, $\iota(G, C_4) \leq (m+1)/6$ and characterized extremal graphs.

These results deepen our understanding of isolation in graphs and suggest intriguing directions for future research.

## References

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