GENERALIZED COPPER FIBONACCI SEQUENCES AND THEIR POLYNOMIALS

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In this study, we introduce a new generalization of the Fibonacci sequence, which we call the *Copper Fibonacci sequence*, that converges to the copper ratio. Also, drawing inspiration from the definition of the Copper Fibonacci sequence, we the *Copper Lucas sequence* and investigate the relationships between the terms of both sequences. We examine several properties of these sequences, including Binet-like formulas and generating functions. In addition, we explore the relationship between the roots of the characteristic equation of these sequences and their general terms. Interestingly, the relationships derived from the connection between the roots and the terms of these new sequences hold true for both roots. Moreover, we introduce the application of these sequences to polynomials. We examine the relations between the terms of the Copper Fibonacci and Copper Lucas polynomials and two consecutive terms. Lastly, we derive special identities associated with these polynomials.

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