HAMILTON PATHS AND CYCLES IN DENSE GRAPHS

Nikola Jedličková, Jan Kratochvíl

Charles University, Prague, Czech republic

e-mail: honza@kam.mff.cuni.cz, jedlickova@kam.mff.cuni.cz

The complexities of both Hamiltonian path and Hamiltonian cycle problems have been open for graphs of independence number at most 3 until recently. We answer these questions in the general setting of graphs of bounded independence number by showing that both these problems are in the class XP when parameterized by the independence number of the input graph.

We introduce a problem called *Hamiltonian-ℓ-Linkage* which is related to the notions of a path cover and of a linkage in a graph. This problem asks if ℓ given pairs of vertices in an input graph can be connected by disjoint paths that together traverse all vertices of the graph. For $\ell = 1$, Hamiltonian-1-Linkage asks for existence of a Hamiltonian path connecting a given pair of vertices. Our main result reads that for every pair of integers k and ℓ, the Hamiltonian-ℓ-Linkage problem is polynomial time solvable for graphs of independence number at most k [2].

A tool used for designing this algorithm is the Chvátal-Erdős theorem on Hamilton connectedness of graphs of large connectivity with respect to the independence number [1], and its generalization to Hamiltonian linkages that we formulate and prove.

The running time estimate we get from our general algorithm is highly exponential in k and ℓ. For small values of k, we describe concrete obstacles for the existence of Hamiltonian cycles and paths in graphs of independence number at most k [3]. Even brute force checking of these obstacles yields algorithms with much better running times than that of the general one.

References

[1] V. Chvátal, P. Erdős, *A note on Hamiltonian circuits*, Discrete Mathematics 2(2) (1972), 111–113.

[2] N. Jedličková, J. Kratochvíl, *Hamiltonian path and Hamiltonian cycle are solvable in polynomial time in graphs of bounded independence number*, arXiv 2309.09228 (2023).

[3] N. Jedličková, J. Kratochvíl, *On the Structure of Hamiltonian Graphs with Small Independence Number*, in: Combinatorial Algorithms - 35th International Workshop, Proceedings IWOCA 2024, (A. A. Rescigno and U. Vaccaro, eds.), Lecture Notes in Computer Science 14764 (2024), 180–192.