ON PERFECT MULTIPLE DOMINATION IN GRAPHS

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A subset $D \subseteq V(G)$ is called a $(1,2)$-dominating set (or $(1,2)$-DS) if each vertex $v \in V \setminus D$ has one neighbor in D and another vertex in D at a distance of at most 2 from v. Every graph has a $(1,2)$-dominating set.

The notion of proper $(1,2)$-dominating sets was introduced by Michalski in [1] and further studied in [2], refining the concept by excluding cases that are $(1,1)$-dominating sets. A subset $D \subseteq V(G)$ is a proper $(1,2)$-DS if D is a $(1,2)$-DS and not a $(1,1)$-DS. A proper $(1,2)$-DS only requires the existence of at least one vertex outside D that satisfies the domination condition, meaning that there exists a vertex $u \in D$ such that $d_G(x,u) = 2$. A graph has a proper $(1,2)$-DS if and only if it is not complete.

The problem becomes more complicated if we imposes this condition on all vertices outside D, studying perfect $(1,2)$-DS. Every perfect $(1,2)$-DS is a proper $(1,2)$-DS, but not every proper $(1,2)$-DS is perfect, as it may fail to enforce this condition for all vertices outside D.

In this talk, we discuss the existence of perfect $(1,2)$-DS in special classes of graphs. Moreover, we consider similar concept for multiple dominating sets.

References

[1] A. Michalski, *Secondary dominating sets in graphs and their modification*, In Book of Abstracts, The 7th Gdańsk Workshop on Graph Theory (2019).

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