

ISOLATION OF 3-VERTEX PATHS

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The P_3 -isolation number of a connected n -vertex graph G , denoted by $\iota(G, P_3)$, is the size of a smallest subset D of the vertex set of G such that the closed neighbourhood $N[D]$ of D in G intersects each P_3 -copy in G (equivalently, no two edges of $G - N[D]$ intersect). The concept of \mathcal{F} -isolation for a general set \mathcal{F} of graphs was first introduced by Caro and Hansberg [2]. The sharp upper bound $\iota(G, P_3) \leq 2n/7$ for a connected n -vertex graph G which is not a $\{P_3, K_3, C_6\}$ -graph was established by Zhang and Wu [5], and independently by Borg [1] in a stronger form.

The graphs attaining the bound $2n/7$ have been determined in [3] and [4]. Infinitely many of these graphs have induced 6-cycles. Our investigation focuses on how the upper bound on $\iota(G, P_3)$ improves upon considering connected graphs G which do not have induced 6-cycles. The resulting sharp upper bounds on $\iota(G, P_3)$ obtained along with some of the resulting extremal structures are some of the first of their kind in the current study of \mathcal{F} -isolation numbers of graphs.

References

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